Analytical and finite-element modelling of the incudostapedial joint

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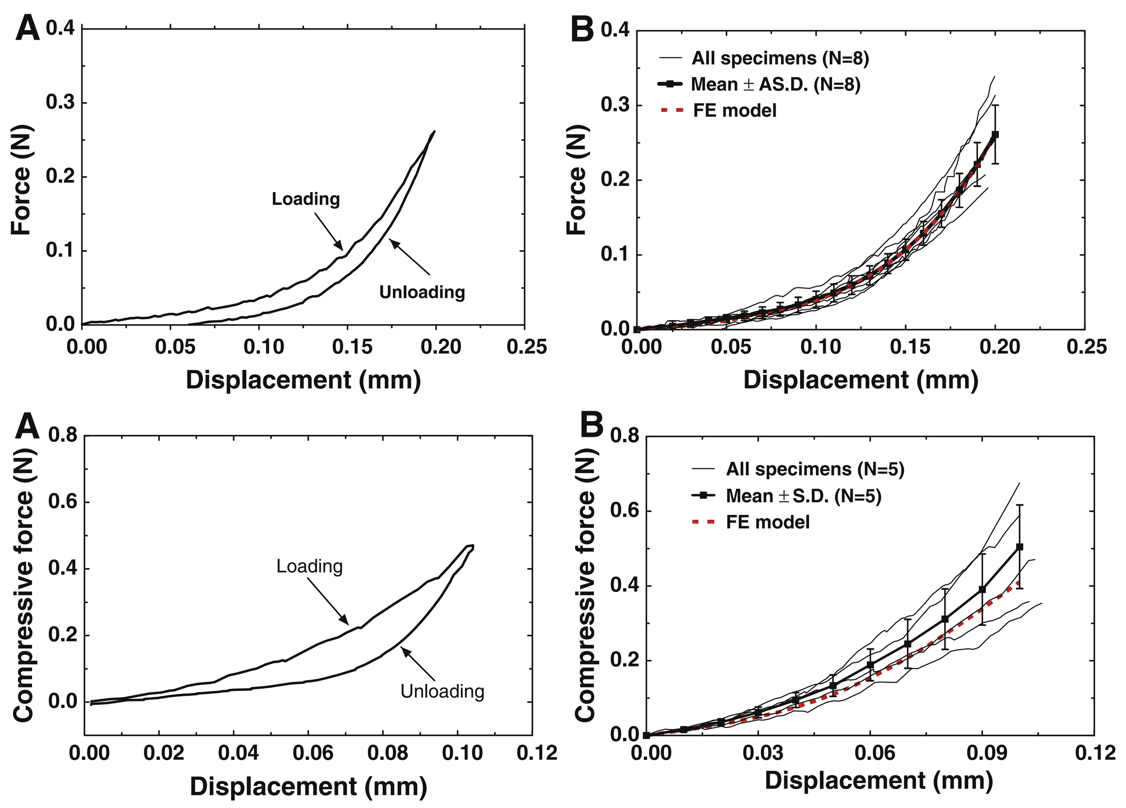
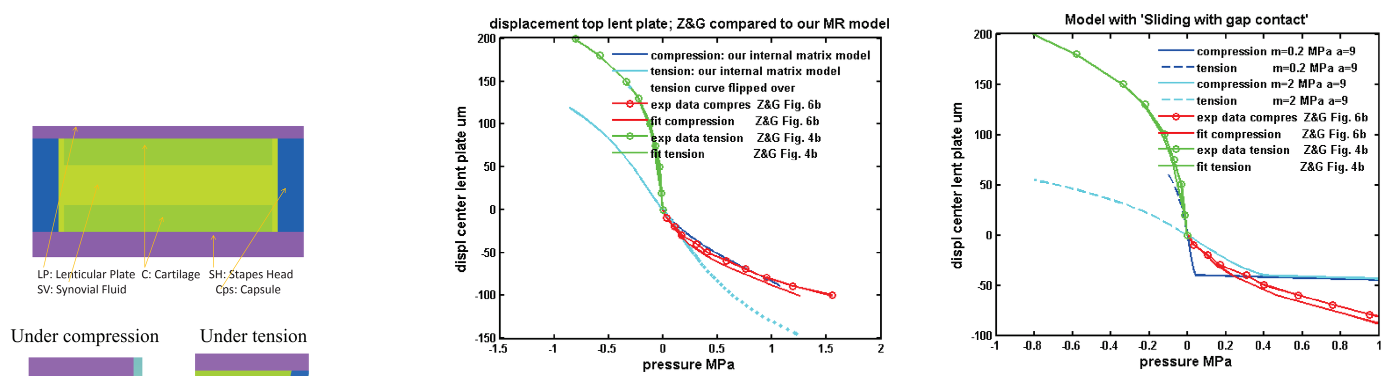
Introduction

- The joints of the ossicular chain significantly affect sound transmission through the middle ear, but the mechanical behaviour of these joints is not well understood. We previously analyzed the incudostapedial joint using a finite-element (FE) model with a simplified geometrical description of the pedicle of the lenticular process, based on histological serial sections and X-ray micro-CT, and using a priori estimates for material-property parameters, but we neglected the presence of synovial fluid (SF) in the joint (Funnell et al., 2005). We later included SF in a FE model and made preliminary comparisons with experimental tension and compression measurements (Decreamer et al., MEMRO 2015).
- Incudostapedial joint (ISJ) - a synovial joint

- Experimental measurements on human incudostapedial joints under tension or compression (Zhang & Gan, 2011, Figs 4 & 6)



- Note the hysteresis (not modelled here) and strong asymmetry
- Our FE simulations done using FEBio (presented previously at MEMRO 2015, Aalborg) compared with Zhang & Gan model

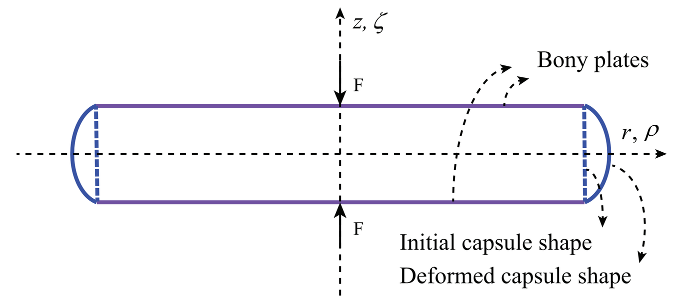


Left: Slightly compressible synovial fluid (Mooney-Rivlin material).
Right: Incompressible synovial fluid with various capsule parameters & thin synovial gap.

- The strong asymmetry in the experimental data is not seen in our model, and the solution sometimes fails to converge

Analytical model with greatly simplified geometry

- Synovial fluid and cartilage on incus and stapes bone modeled as thin cylindrical slab with circular cross section
- ISJ capsule approximated as thin cylindrical tube surrounding the slab
- we use the theory of large deformations of elastic membranes to analytically model ISJ



- we use the theory of large deformations of elastic membranes to model the capsule
- Core filled with incompressible (fluid-like) Mooney-Rivlin material with strain energy function

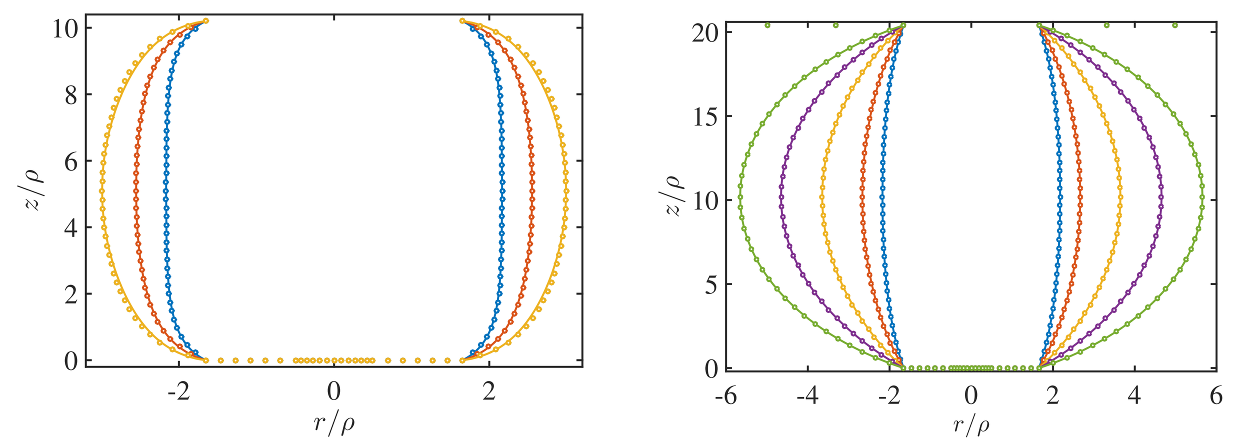
$$W = C_1(I_1 - 3) + C_2(I_2 - 3)$$

- Two cases studied: Fixed bony plates with varying volume and constant volume with varying pressure
- Parameters: Λ_1: local stretch ratio in z direction at $z = 0$; $\Lambda_2 = r/\rho$: local stretch ratio in θ direction (inflation) at $z = 0$ P: internal pressure; C_1 & C_2: Mooney-Rivlin coefficients; l_0: initial membrane length.

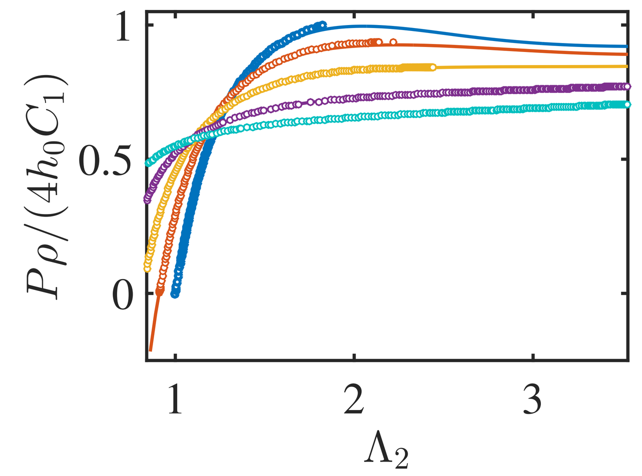
Results

- Comparing analytical solutions (MATLAB) and FE solutions (FEBio)

Note that these FE models are different from the previous incudostapedial joint model. These are cylindrical membranes with the same geometry as the analytical model.

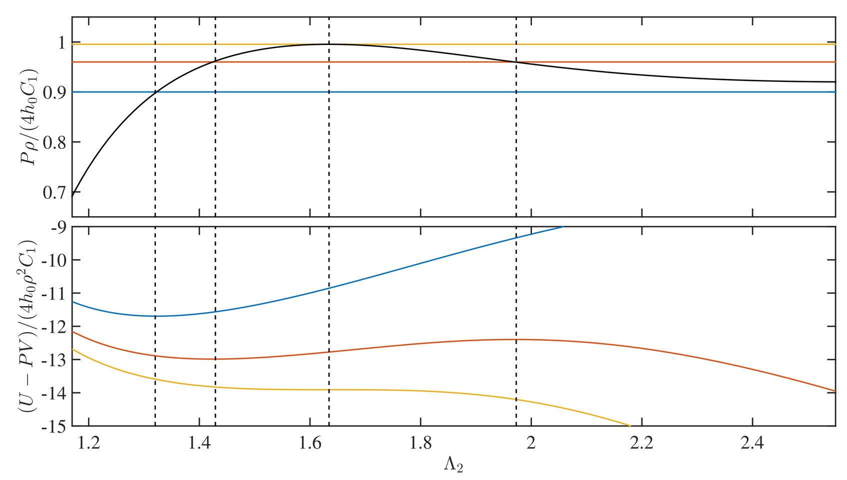


Tube profiles from analytical solution (solid lines) compared with FE calculations (symbols) at various inflations when bony plates are fixed without initial elongation (left) and with initial elongation (right).



Pressure vs inflation at the centre with fixed walls and various initial elongations. The FE solution fails to converge when reaching the peak pressure because of instability. When the instability is removed by prestretching the membrane, the FE solution converges.

- Stability analysis



Top: Typical pressure-vs-inflation behaviour where equilibrium configurations at various pressure values are intersections of constant-pressure lines with pressure equilibrium path. Bottom: Energy of the same constant-pressure configurations showing whether the equilibriums are minima or maxima of energy. The maximum pressure limit corresponds to inflection points in the energy curves, meaning that the equilibriums are stable only before reaching the maximum pressure.

Acknowledgement

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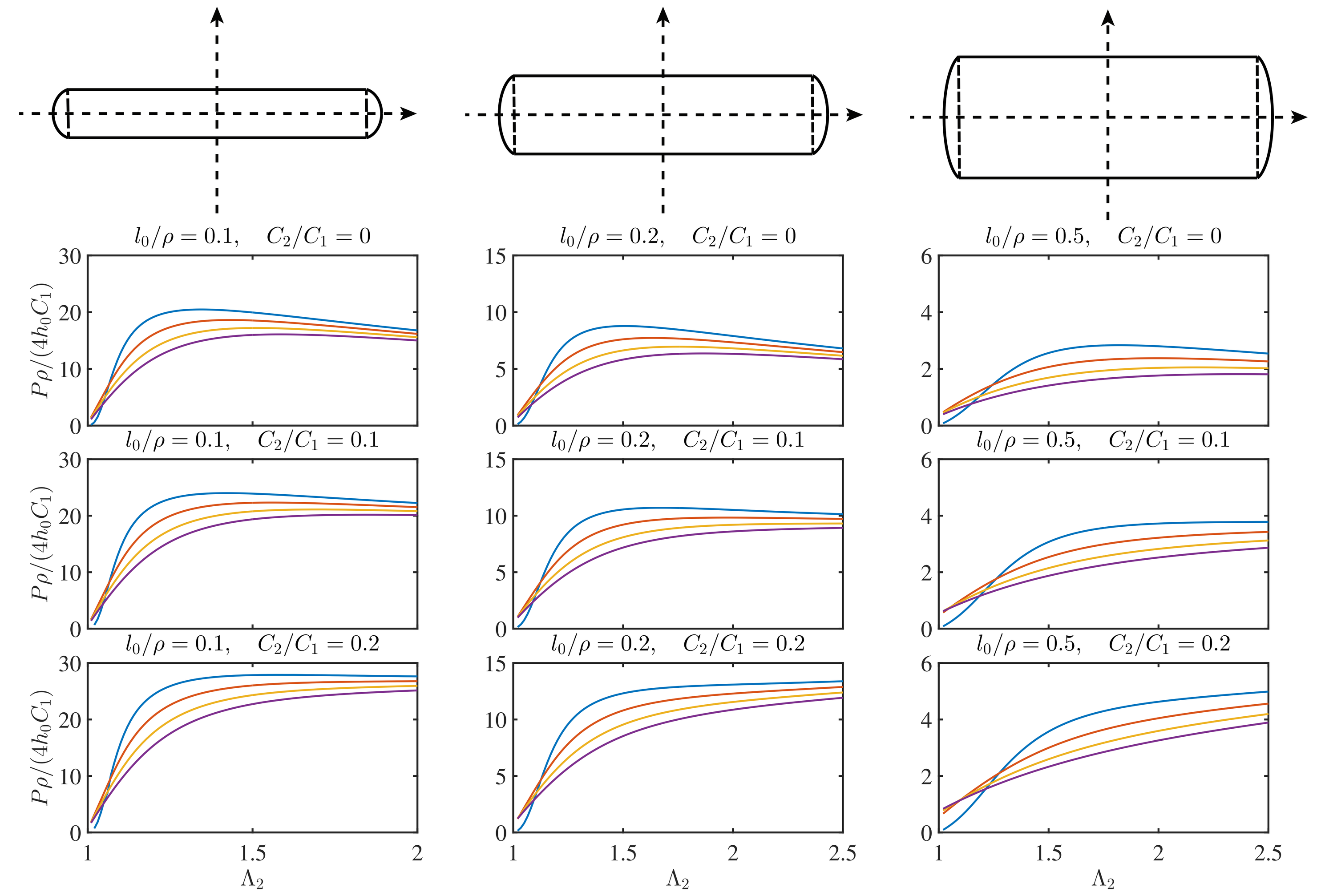
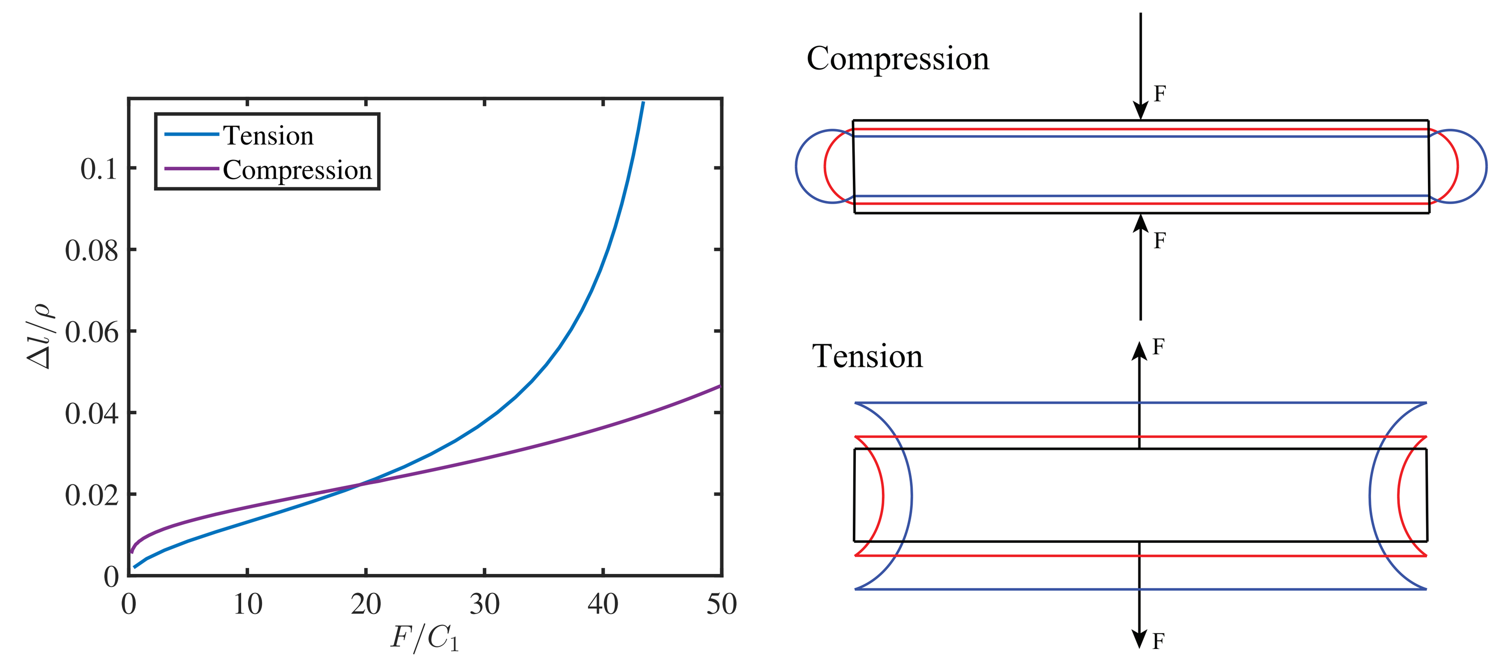
Results (cont'd)

- Sensitivity analysis with fixed bony plates



Pressure vs inflation when changing initial elongation, membrane length, and elastic constants. The membrane length is increased from left to right and C_2/C_1 is increased from top to bottom. In each graph, the initial elongation is increased from blue lines to purple lines. Increasing length, C_2/C_1, and initial elongation stabilizes the membrane.

- Fixed volume:



Membrane elongation vs force when maintaining constant volume, showing the asymmetry between tensile and compressive loads.

Conclusions

- We used simplified analytical and FE models to analyse the mechanical behaviour of the incudostapedial joint
- The mechanical behaviour of the joint may be more complicated than expected, with instabilities involved when increasing the internal pressure
- The instabilities can be removed by prestretching the membrane and using longer and stiffer membranes
- Although the behaviours of the analytical and FE models are not symmetrical for compressive and tensile loads, the strong asymmetry in the experimental measurements is not seen yet

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