Abstract #48 Simulating large deformations of the gerbil pars flaccida to determine its material properties

Wim F. Decraemer[1*], Joris J.J. Dirckx[1], Nima Maftoon[2], W. Robert J. Funnell[2,3]

[1]Biomedical Physics, University of Antwerp, Belgium, [2]Department of BioMedical Engineering and [3]Department of Otolaryngology - Head and Neck Surgery, McGill University, Montréal, Canada.

ABSTRACT

The gerbil is a popular animal in hearing research and many aspects of its middle-ear function have been studied. Various papers have reported on studies of its pars flaccida in the last decade. For example, the role of the pars flaccida in low-frequency hearing (Teoh et al., Hear. Res. 1997; Rosowski et al., Audiol. Neuro-Otol. 1999) was investigated and the deformation of the pars flaccida with static pressure in healthy animals and in animals with otitis media (Dirckx et al., Hear. Res. 1997, 1998) was studied. The interpretation of these results and hence a better understanding of the fine details of the functioning of the hearing organ can benefit greatly from mathematical models in which structural elements of the ear are represented in a geometrically correct way and with correct physical characteristics. Finite-element analysis is often used for such mathematical models but there has been relatively little modeling of middle-ear deformations that exceed the linear (infinitesimal) range. The pars flaccida in gerbil has a flat and nearly circular shape, and in recent studies in our laboratory the thickness has been determined (Kuypers et al., JARO 2005). This makes the design of a realistic finite-element model for the gerbil pars flaccida relatively straightforward, but the material properties are not known. We have used both linear elastic and hyperelastic formulations for the constitutive equations and adjusted their parameters to find simulations that replicate well the large 3D deformations of the gerbil pars flaccida measured in our lab.

ANATOMY OF THE PARS TENSA AND PARS FLACCIDA IN GERBIL

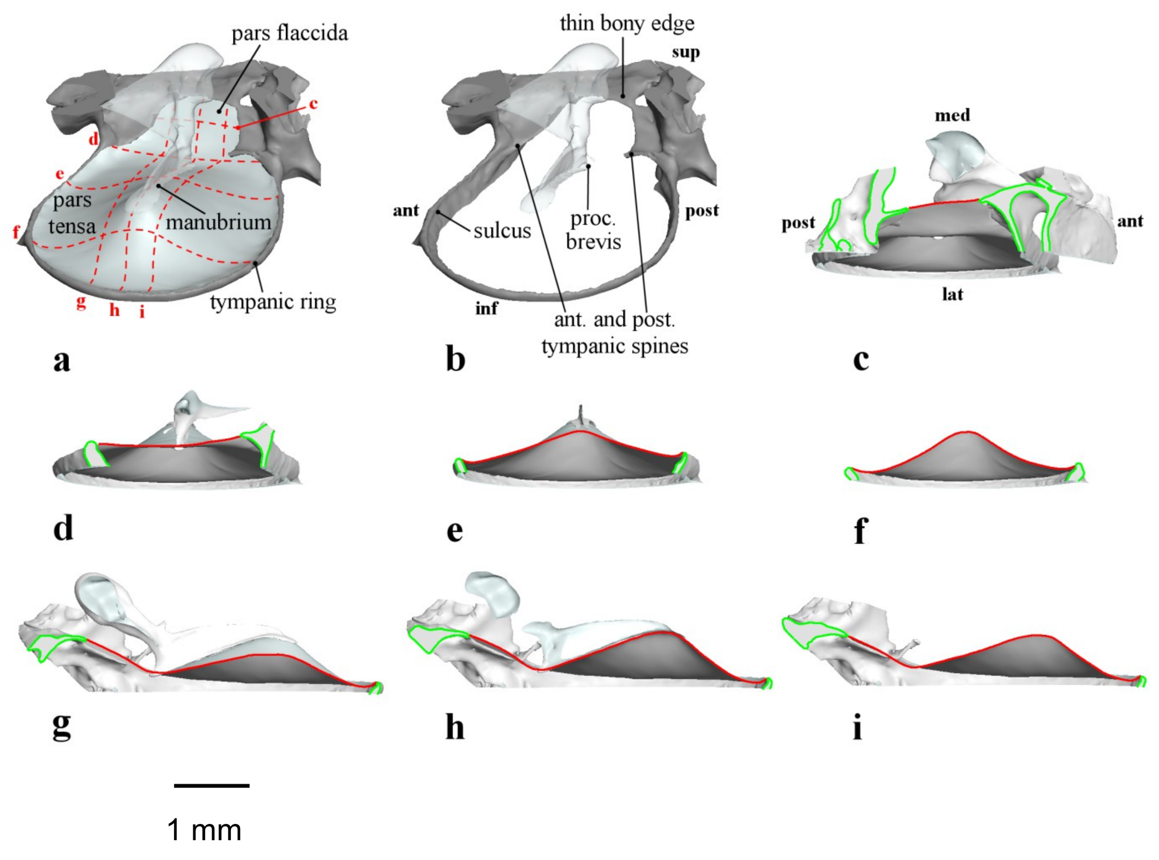
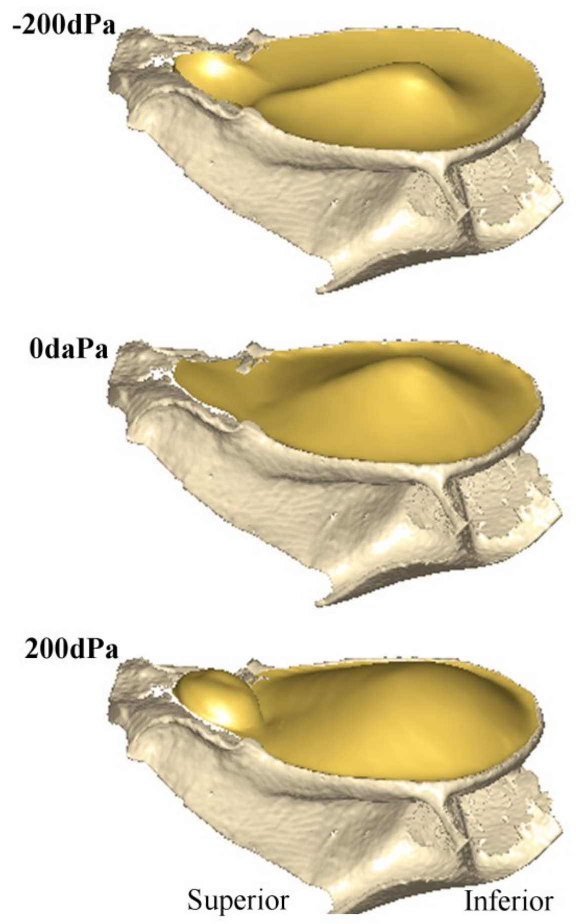


Fig. 1: The gerbil has a relatively large and nearly circular pars flaccida. It is bounded anteriorly, superiorly and posteriorly by a horseshoe shaped, thin bony ledge. The inferior boundary is a crease between the pars tensa and pars flaccida, reinforced by the anterior and posterior striae.

UNDERS STATIC PRESSURE THE PARS FLACCIDA DEFORMS INTO A NEARLY PERFECT SPHERE CAP



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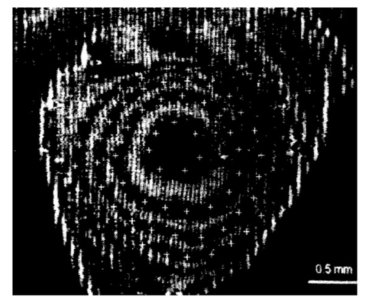

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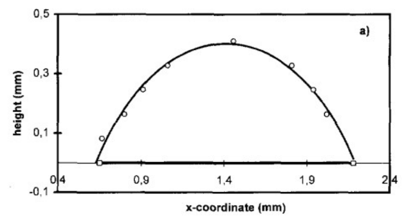


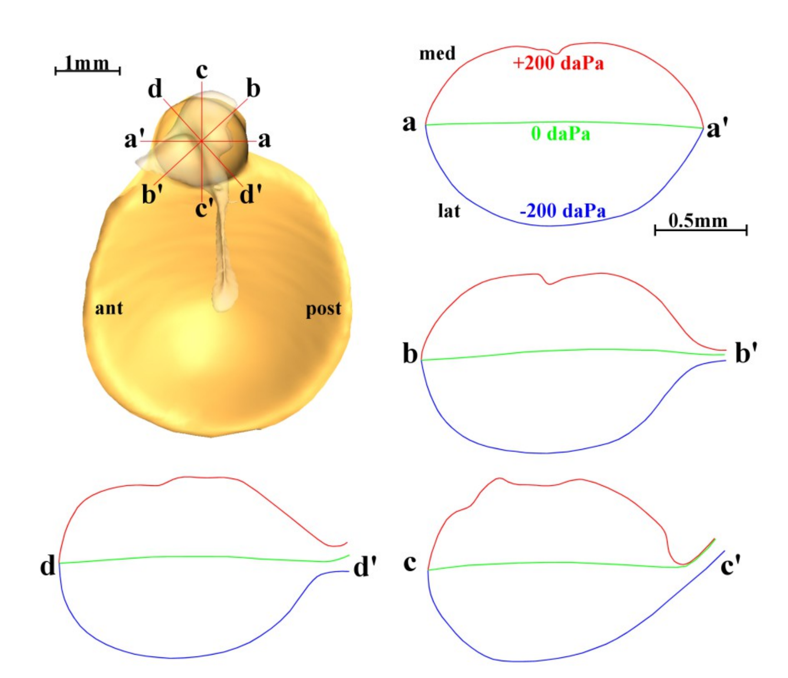
Circle fit to cross-section through center of a pressurized pars flaccida

Fig. 2a: 3D pars flaccida deformation based on micro-CT

Fig. 2b: Pars flaccida deformation measured with moiré interferometry

BOUNDARY CONDITIONS FOR THE PARS FLACCIDA

Fig. 3: Cross-sections of the pressurized pars flaccida based on micro-CT



At the edges with the bone the pars flaccida has a hinge-like deformation: this indicates that a *simply supported mathematical boundary condition* is appropriate. At the crease with the pars tensa the situation is not so clear-cut.

MATHEMATICAL MODEL FOR THE PARS FLACCIDA

The above indicates that we can model the pars flaccida as

- a flat, circular thin shell (radius of 0.7 mm, estimated from moiré recordings)
- with a simply supported boundary condition around its entire boundary.
- As a first step we disregard thickness variations and choose a uniform thickness of 25 micron (average value for the thinner central zone as measured by L. Kuypers , 2005)

CHOICE OF MATERIAL DESCRIPTION

The pars flaccida is a soft biological tissue. Typical features of stress-strain curves for soft biological tissue include: shallow onset, then a zone with rapid increase of the slope, and an approximately linear end part for large strains. Over a large range of strains the material is clearly not linear elastic and in a finite-element model it must be represented as a hyperelastic material. We tested a few candidate constitutive equations that are implemented in the two free finite-element packages that we used and that support biological tissues: FEBio (Dept.of Bioengineering, University of Utah) and CalculiX (Guido Dhondt and Klaus Wittig, MTU Munich).

1/Curve-fitting models:

Ogden

$$\sigma_{Og}(\lambda,\alpha_1,\mu_1) := 2\cdot\frac{\mu_1}{\alpha_1}\cdot\left[\lambda^{\alpha_1-1} - \lambda^{-(0.5\cdot\alpha_1+2)}\right]$$

Veronda-Westmann

$$\sigma_{VW}(\lambda,C_1,C_2) := 2\left(\lambda^2-\frac{1}{\lambda}\right)\cdot(C_1\cdot C_2)\cdot\left[e^{C_2\cdot\left(\lambda^2+\frac{2}{\lambda}-3\right)}-\frac{1}{2\lambda}\right]$$

2/ Structural model: *W. Decraemer (1980)*

$$\sigma_D(l) := \frac{b}{\sqrt{2\cdot\pi}\cdot s}\cdot\int_{l_0}^{l}\frac{1-l_i}{l_i}\cdot\left[\exp\left[-\frac{(\mu-l_i)^2}{2\cdot s^2}\right]\right]dl_i$$

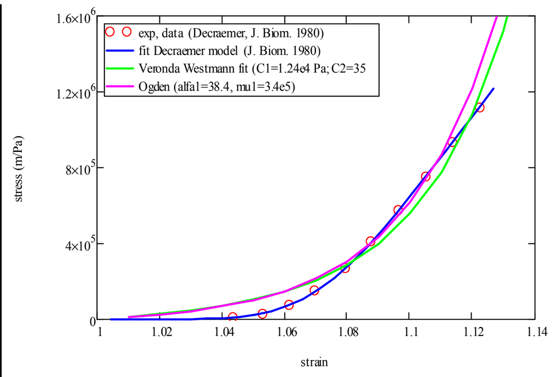


Fig. 4: Three hyperelastic equations fitted to a stress-strain relation for human pars tensa

FINITE-ELEMENT MODEL FOR THE GERBIL PARS FLACCIDA

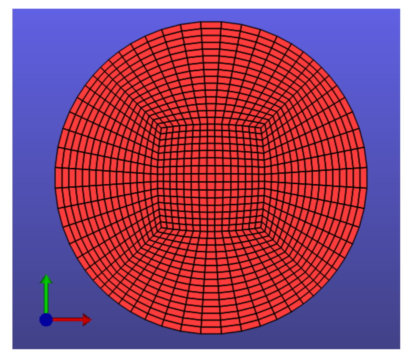


Fig.5: Circular finite-element model for the gerbil pars tensa meshed with quadrilateral shell elements

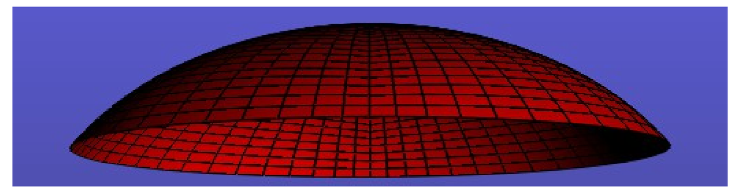


Fig. 6: Under a positive middle-ear pressure of 2 kPa the initially flat pars flaccida deforms into a nice bowl shape

FITTING TWO MODELS TO EXPERIMENTAL DEFORMATION DATA

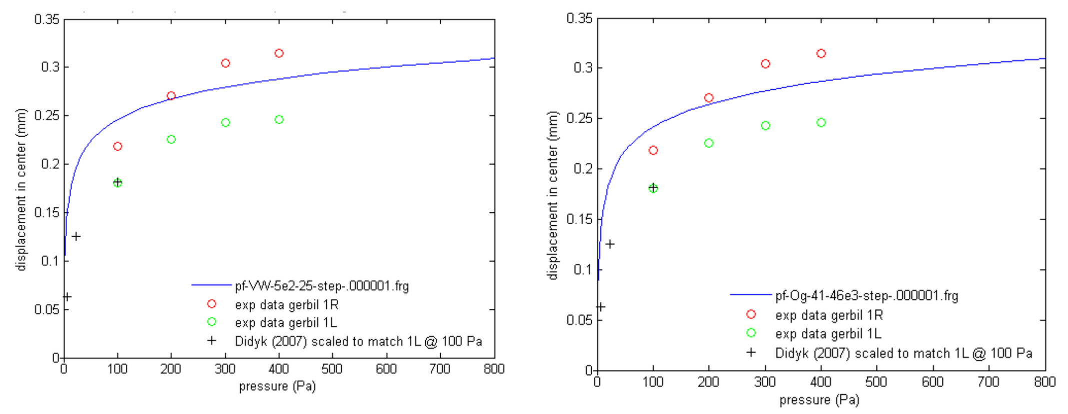


Fig. 7: Displacement at the center of the pars flaccida as a function of pressure for a few different animals (symbols) and a fit (solid line) with a Veronda-Westmann shell model (left panel, FEBio simulation) and an Ogden model (right panel, CalculiX simulation)

Strong initial rise and final leveling off of the displacement curve can be obtained by both models

Remark also inter-animal variability in the experimental data

Model choice more often based on ease of implementation (e.g. ease of convergence)

GEOMETRICAL NON-LINEARITY FOR A LINEAR ELASTIC MATERIAL

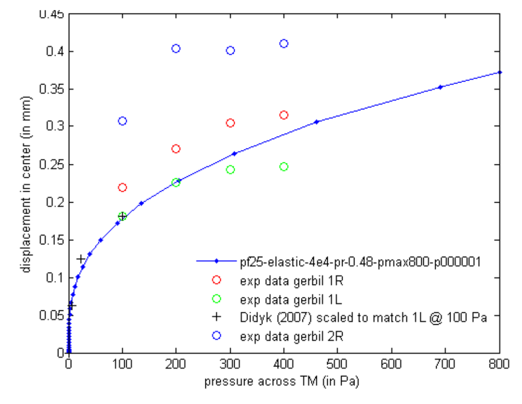
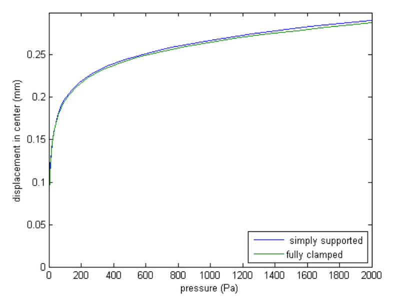


Fig.8: Displacement at the center of the pars flaccida as a function of pressure for a few different animals (symbols) fitted with a linear elastic material (CalculiX simulation)

Even with linear elastic material properties we do obtain a non-linear displacement as a function of pressure, due to geometrical non-linearity. However, the curve does not level off sufficiently at high pressures to fit the experimental data well.

EFFECT OF BOUNDARY CONDITIONS FOR LARGE PRESSURE (2 kPa)



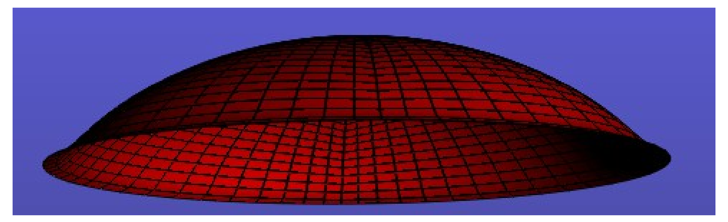


Fig. 9: When in the model used for Fig. 6 the boundary conditions are changed from simply supported to fully clamped, the center displacement is not significantly altered for large pressures (right); the deformation at the edges changes as expected (left).

EFFECT OF BOUNDARY CONDITIONS FOR A SMALL (ACOUSTICAL) PRESSURE (0.025 Pa)

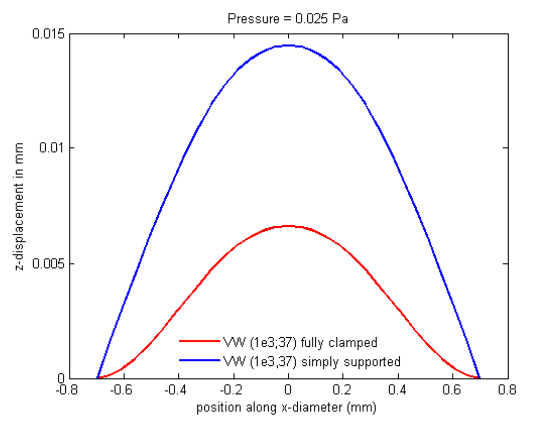


Fig. 10: For pressures in the acoustical range the deformation is strongly dependent on the chosen boundary condition. This is illustrated for a pars flaccida model with Veronda-Westmann material loaded with 0.025Pa.

0.025 Pa_{rms} corresponds to 59 dB SPL

SIMULATION OF THE EFFECTS OF VARIABLE THICKNESS

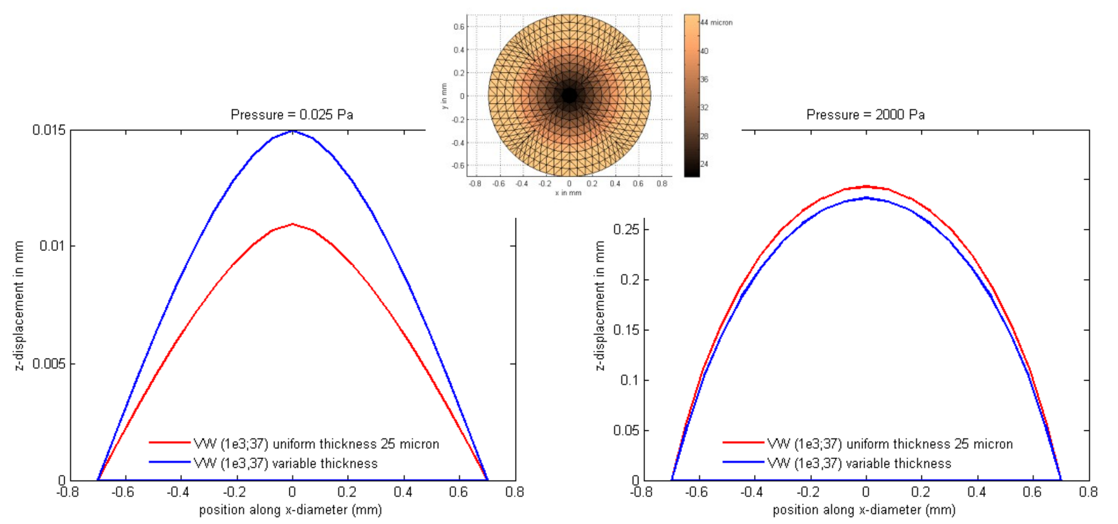


Fig. 11: Making the pars flaccida model gradually thicker in a band around the edge (a gross approximation of the real, less systematic, local thickness variation) decreases the deformation and changes the deformation profile. The effects are again more pronounced for low pressure loads. See insert for the thickness used in the model.

REFINING THE MODEL AT THE PARS TENSA/PARS FLACCIDA CREASE

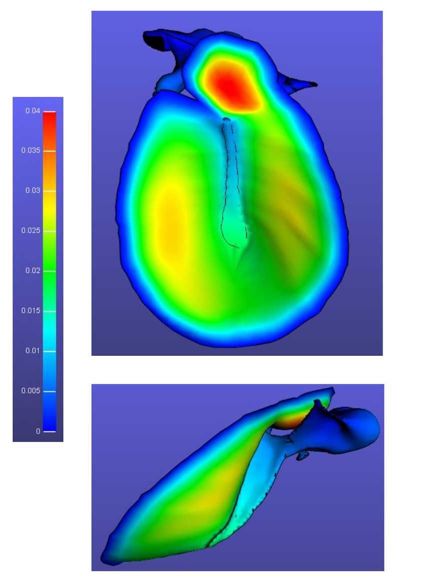
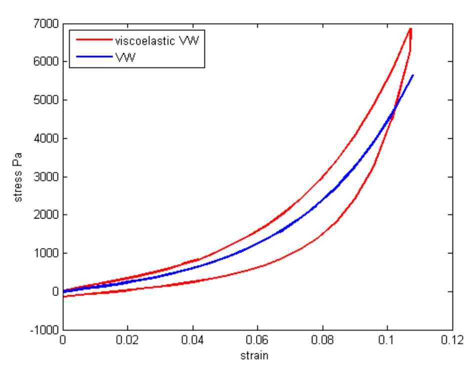


Fig. 12: In a more complete model of the gerbil ear based on CT measurements of a gerbil temporal bone, the border between the pars tensa and pars flaccida is not constrained to be fixed. This looser boundary part causes the deformation of the pars flaccida to become somewhat asymmetric. In the finite-element simulation shown here, the pattern is stretched out in the area near the posterior stria (see upper panel). The lower panel shows again the cup-like bulging of the pars flaccida for earcanal overpressure.

VISCOUS EFFECTS



In a following we step we have to analyze how the deformation of the PF changes with time when viscoelastic effects are introduced. In FEBio this is done by multiplying the constitutive equation by a suitable relaxation function.

Fig. 13: Stress-strain relation for strip of Veronda-Westmann material, and for a viscoelastic version of the same material. The latter displays the typical hysteresis. (VW parameters of Fig. 11).

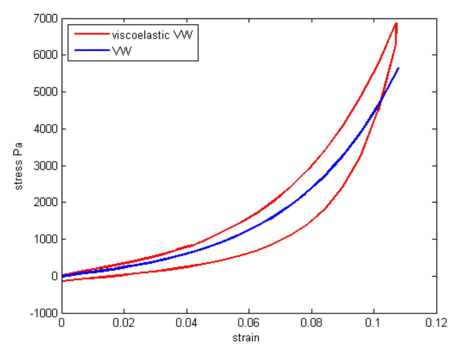


Fig. 14: Displacement at the center of the pars flaccida for a Veronda-Westmann with added viscous effects when subject pressure to a saw tooth pressure cycle. (VW parameters of Fig. 11).

REFERENCES

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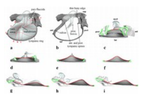

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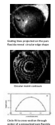

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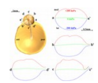

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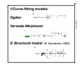

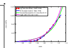

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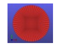

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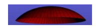

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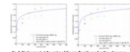

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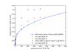

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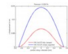

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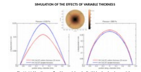

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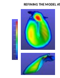

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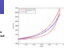

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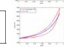

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